Lab 11/5

October 31, 2019

Hypothesis Testing

A good approach to statistical analysis is to have an analysis plan in mind before you look at the data. What procedure would you use to analyze the following scenarios?
For each problem, please include
the name of the statistical test
the parameter being tested, value of the parameter, and the signs of H_0 and Ha.

Example: What test would you use to determine whether the starting salaries for statisticians are greater than $80,000?
Answer: One sample t-test for population mean, $H_0 : \mu = 80,000$ vs $H_a : \mu > 80,000$.

1. For a newly developed rose, a florist measures the diameter of 100 flowers in full bloom and finds 82 with diameters in excess of 4.5 inches. What statistical procedure would you use to test whether the true proportion of flowers with diameters greater than 4.5 inches is more than 80% at $\alpha = 0.10$?

   One sample z-test for population proportion, $H_0 : p = 0.8$ vs. $H_0 : p > 0.8$.

2. The average daytime temperature for December 2018 as recorded at the Minneapolis-St. Paul Inter- national Airport was 31.6 F°. Historically, the daytime temperature was 27.1 F°for the month of December at the MSP airport. What statistical procedure would you use to test whether there is any evidence of global warming at $\alpha = 0.05$?

   One sample t-test for population mean, $H_0 : \mu = 27.1$ vs. $H_a : \mu > 27.1$.

Multiple Choice

For each problem, choose the correct answer and explain your choice.

3. If H_0 is rejected at $\alpha = 0.05$, which of the following statements are always true? i H_0 would be rejected at $\alpha = 0.01$
ii H_0 would be rejected at $\alpha = 0.10$
iii both i. and ii.

ii is true, because the p-value was less than 0.05 which would also be less than 0.1. It is unknown if the p-value is less than 0.01.

4. If the 99% confidence interval for μ does NOT include μ_0, which of the following statements are always true?
i the 90% confidence interval would contain μ_0
ii the 95% confidence interval would NOT contain μ_0
iii in a two-sided test, H_0: $\mu = \mu_0$ would NOT be rejected at $\alpha = 0.05$
iv in a two-sided test, H_0: $\mu = \mu_0$ would NOT be rejected at $\alpha = 0.10$

ii is true, because a 99% CI is wider than a 95% CI, which implies i is false. iii and iv are false because this confidence interval implies a two-sided test would reject H_0 at level $\alpha = 0.01$.

R Problem

1. Alf Alpo is a Quality Control Engineer for a dog food company. His job is to monitor the batches of dog food at the manufacturing plant to ensure that it meets the formula as stated on the packages label. Everyday Alf randomly samples 25 batches of food and analyzes the nutrient content of the dog food. One of the ingredients in the dog food is glucosamine, which promotes joint health in older dogs. The plant machine is set to deliver 475 mg/kg of glucosamine. Alf uses his data to statistically test whether the machine is hitting its target. Data for 20 consecutive days was collected. He is interested in testing the following hypotheses:

$$H_0 : \mu = 475 \text{ vs. } H_a : \mu \neq 475$$

Run the first set of R code to obtain 20 p-values.

- Based on these results, how many times out of 20 did you reject H_a if $\alpha = 0.05$?

One time.

- Based on these results, how many times out of 20 did you reject H_0 if $\alpha = 0.01$?
No times

- How do these results represent type I error?
A type I error is rejecting the null hypothesis when the null is true. Here we set the null hypothesis to be true and we saw that 2 times out of 20, we made a type I error. If we were to repeat this more times, we would see the proportion of times we rejected should approach $\alpha = 0.05$.

2. Alf retired and Cruella de Vil took his position. She was not as careful as Alf and did not monitor the dog food formula on a daily basis. The machines accuracy was not checked and it drifted out of specifications (unknown to Cruella). When her manager found out, he asked her to sample the dog food for a single day to check the machines accuracy.

   Use the data in part 2 of the script and t.test() command in R to determine if it was delivering 475 mg/kg of glucosamine at $\alpha = 0.05$. Include all 5 steps of the hypothesis test.

   i. Assumptions: Random sample - assumed, data come from a normal distribution - can check with a qqplot.
   ii. Hypotheses: $H_0 : \mu = 475$ vs. $H_a : \mu \neq 475$.
   iii. Test stat: $T = 2.614$
   iv. P-value: use `t-test(glucosamine, alternative = 'two.sided', mu = 475)`
   v. Conlcusion: p-value was 0.015, which is less than 0.05, so we reject the null hypothesis that the machine was delivering 475 mg/kg of glucosamine.

3. Cruella de Vils manager was not happy with the test results from a single day (part 2) and asked her to collect data for 20 more days and analyze it in the same manner that Alf did.

   Run the last part of the script to obtain the 20 p-values and 95% confidence intervals. Explain how the results of the 95% confidence intervals agree with the test of $H_0 : \mu = 475$ vs. $H_a : \mu \neq 475$ based on α=0.05.

   We see that when the p-value is less than 0.05, the CI does not contain 475 and when the p-value is greater than 0.05, the CI contains 475.