# Lab week 9 Solutions

## Problem 1

Supposed we want to test how random a coin flipping simulator is. In order to test this, we run the simulator 50 times and record the number of heads.

1. Write out the null and alternative hypotheses corresponding to this research question. **Solution:** $H_0 : p = 0.5$ vs. $H_a : p \neq 0.5$.

2. In one sample of 50, suppose we obtained 30 heads. Peform the five steps of the hypothesis test at level 0.05 corresponding to the hypotheses in 1. State the conclusion clearly about the how random this simulator is.

   **Solution:**

   1. Assumptions: atleast 15 expected successes and failures? $50*0.5 = 0.25$, yes.
   2. Hypotheses: $H_0 : p = 0.5$ vs. $H_a : p \neq 0.5$.
   3. Test Statistic: $z = \frac{0.6-0.5}{\sqrt{0.25/50}} = \frac{0.1}{0.0707} = 1.414$.
   4. p-value: $2P(T > |1.414|) = 2(1 - \texttt{pnorm(1.414)}) = 0.157$
   5. Interpretation and Conclusion: The p-value says under the null hypothesis the probability we obtained 30 heads or more extreme is 0.157, indicating our results do not stray too far from the null hypothesis. In addition, our p-value was greater than the level, so we would fail to reject the null hypothesis that the simulator is random.

3. Perform this test in R using `prop.test()` function.

   **Solution:**

```
prop.test(x = 30,n=50,p=0.5,alternative='two.sided',conf.level = 0.95,correct = FALSE)
```

```
##
##  1-sample proportions test without continuity correction
##
## data:  30 out of 50, null probability 0.5
## X-squared = 2, df = 1, p-value = 0.1573
## alternative hypothesis: true p is not equal to 0.5
## 95 percent confidence interval:
##  0.4618144 0.7239161
## sample estimates:
##   p
## 0.6
```

4. Using the R output, provide a 95% CI for $p$. Combine with the hypothesis test, what do you observe?

   **Solution:** The confidence interval contains 0.5 which would be the proportion of heads that a truly random simulator would generate. This interval is consistent with our conclusion.

5. Suppose instead we believed the simulator flipped more heads than tails. Redo the hypothesis test starting at 1, what has changed?

   **Solution:** The only thing that has changed are the p-value and hypotheses. The new hypotheses are: $H_0 : p = 0.5$ vs. $H_a : p > 0.5$, the new p-value is: $P(T > 1.414) = (1 - \texttt{pnorm(1.414)}) = 0.078$. And because the p-value is greater than 0.05 we would still fail to reject the null hypothesis that the simulator is random.

## Problem 2

A 2006 study considered whether dogs could be trained to detect lung or breast cancer by the smell of a subject's breath. Five ordinary household dogs were trained to distinguish, by scent alone, exhaled breath samples of 55 lung cancer patients and 31 breast cancer patients from those of 83 control patients. Once trained, researchers tested the dogs' abilities to detect cancer on a new set of samples not previously encountered by the dogs. They had each dog try to identify the one cancer sample (where a sample is exhaled breath) from a set of five samples; one of which was from a cancer patient, four of which were from controls. Let p denote the probability a dog correctly detects the cancer sample from the five samples.

1. Write out the null and alternative hypotheses corresponding to whether the dog's prediction is better than random guessing in the researcher's test. **Solution:** $H_0 : p = 0.2$ vs. $H_a : p > 0.2$

2. In one sample of 83, (where a set is the five patients, one of whom had cancer), the dogs correctly identified the cancer sample 81 times. Perform the five steps of the hypothesis test corresponding to the hypotheses you stated in the previous part. Use $\alpha = 0.05$. State your conclusion clearly about the dog's ability to detect cancer. **Solution:**

   1. Assumptions: expected successes and expected failures under $H_0$ are at least 15. 0.2*83 = 16.6 > 15, 0.8*83 = 66.4 > 15.
   2. Hypotheses: $H_0 : p = 0.2$ vs. $H_a : p > 0.2$
   3. Test Statistic: $z = \frac{81/83-0.2}{\sqrt{0.2*0.8/83}} = 17.67$.
   4. p-value: $P(Z > z) = P(Z > 17.67) = 1 -$ `pnorm(17.67)` $= 0$.
   5. Conclusion: The p-value says the probability of randomly guessing 81 or more out of 83 samples correctly is $0 < 0.05 = \alpha$. We reject the null hypothesis the dogs are randomly guessing in favor of the alternative hypothesis the proportion of correct diagnoses by the dogs is greater than 1.

3. Perform this test in R using `prop.test()` function. **Solution:**

From the output below, our conclusion above is consistent with the test in R.

```
prop.test(x=81,n=83,p=0.2,alternative = 'greater')
```

```
##
##  1-sample proportions test with continuity correction
##
## data:  81 out of 83, null probability 0.2
## X-squared = 307.47, df = 1, p-value < 2.2e-16
## alternative hypothesis: true p is greater than 0.2
## 95 percent confidence interval:
##  0.9212496 1.0000000
## sample estimates:
##         p
## 0.9759036
```

4. Using the R output, provide a 95% CI for $p$. Combine with conclusion of the hypothesis test, what do you observe? **Solution:** Based upon the R output above, the 95% CI for $p$ is [0.92,1] which is consistent with the test that $p$ is not 0.2.

5. Suppose instead the dogs correctly identified the cancer sample 20 times. Redo 3 and 4. **Solution:** From the output below, the p-value is greater than 0.05. We would therefore fail to reject the null hypothesis. We cannot conclude the dogs diagnoses are better than guessing. The 95% CI for $p$ is [0.16,1] and 0.2 is contained in this interval which is consistend with the conclusion of the hypothesis test.

```
prop.test(x = 20,n=83,p=0.2,alternative = 'greater')
```

```
##
```

```
##  1-sample proportions test with continuity correction
##
## data:  20 out of 83, null probability 0.2
## X-squared = 0.63328, df = 1, p-value = 0.2131
## alternative hypothesis: true p is greater than 0.2
## 95 percent confidence interval:
##  0.1675172 1.0000000
## sample estimates:
##         p
## 0.2409639
```