STAT 3021
Spring 2019
Exam 1 (A)
Time Limit: 50 Minutes

Name (Print): SOLUTION

Lab Section: _____

Instructions:

- Do *not* begin or turn this page until you are instructed.
- Enter all requested information on the top and bottom of this page, and put your initials on the top of every page, in case the pages become separated.
- This exam contains 9 pages (including this cover page and the multiple choice answer sheet). Check to see if any pages are missing. There are 5 True/False questions and 4 short answer problems with possible sub-questions.
- The exam is closed book. You may *not* use your books, or any wireless device on this exam.
- You may use a calculator and one sheet of paper (size A4 or 8.5" by 11") with formulas or other notes on both sides. You may *not* share calculators or notes!
- Show all your work on each problem for full credit except multiple choice problems. The following rules apply:
  - *Organize your work*, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
  - *Mysterious or unsupported answers will not receive full credit* for short answer problems. A correct answer, unsupported by calculations, explanation, or algebraic work will not receive full credit; an incorrect answer supported by substantially correct calculations and explanation may still receive partial credit.
  - If you need more space, use the back of the pages; clearly indicate when you have done this.

Honesty Statement and Pledge:

I have not given or received any aid or assistance to or from any other student in this course during the exam period. Everything I have written on this exam represents my own work and knowledge. I sign this knowing that infringements on the University's Academic Honest policy may result in failure or expulsion.

Signed By: _____ **Date:** _____

Problem I. Multiple Choice

Choose the ONLY ONE correct answer for each question. Circle your answers to all questions in the answer sheet provided. (NO explanation is needed).

1. (2 points) A single die is rolled and you observe the outcome. Let A be the event the outcome is a prime number(2, 3, or 5). Let B be the outcome is less than 4. What is $P(A \cap B^c)$

*** (A) 1/6
(B) 1/4
(C) 1/3
(D) 1/2

2. (2 points) An insurance company sells a policy to airline passengers. If a flier misses the purchased flight due to medical reasons, the policy gives \$200 to the flier. Otherwise, there is no return. Records show that about 2% passengers miss flight due to illness. You buy the policy for your next flight.
Select the **incorrect** statement.

(A) The amount of money you could receive is either \$0 or \$200.
*** (B) **The expected value of the amount of money you receive is \$100**.
(C) The standard deviation of the amount of money you receive is \$28.
(D) The probability you receive no money from the insurance company is 0.98.

$\mu = 200(0.02) + 0(0.98) = 4$
$\sigma^2 = (200-4)^2(0.02) + (0-4)^2 * (0.98) = 784$**. Hence** $\sigma = \sqrt{784} = 28$**.**

3. (2 points) Which of the following is a correct statement about sample statistic?

(A) Range is resistant to outlier.
(B) Sample variance (s^2) is always positive.
*** (C) **In general when a distribution is right-skewed, mean is larger than median.**
(D) Sample statistic is unknown unless you observe the entire population.

Problem II. Be sure to show all work for full credit.

Based on a survey from Stat 3011 Spring 19, 477 students were asked their position on the legalization of marijuana in Minnesota. A person's gender was asked at the same time. Use the given table to answer the following questions. **Round your answers to two decimal places.**

	Legalizing Marijuana		
Gender	No	Yes	total
Female	102	139	241
Male	56	180	236
Total	158	319	477

1. (2 points) What proportion of the students are either female or favor legalization of marijuana?

(102+139+180)/477=421/477=0.8826

2. (2 points) What proportion of the students favors legalization of marijuana in Minnesota **given that** the student is male?

180/(56+180)=0.7627

3. (3 points) Do you think student's gender and their position on legalization of marijuana are independent? Show your work.

P(favor|female)=139/(102+139)=0.5767, hence 57% of female favors legalization of marijuana.
P(favor|male)=180/(56+180)=0.7627, hence 76% of male favors legalization of marijuana.
Hence male are more likely to favor legalization of marijuana so gender and their response are not independent.
Answer may vary.

Problem III. Be sure to show all work for full credit.

1. (4 points) An art gallery displays 10 distinct paintings in a row. Of these paintings, 2 are by Van Gogh, 5 are by Picasso, 3 by Monet. Find the number of different ways the paintings can be displayed if the paintings by each of the artists are kept together.

There are $3! = 6$ ways to put three artist in order.
For each arrangement, Gogh's can be arranged in 2!=2 different ways, Picasso's can be arranged in $5! = 120$
Monet's can be arranged in $3! = 6$ different ways.
Hence there are $3! \times 2! \times 5! \times 3! = 8640$

2. (4 points) An art gallery displays 10 distinct paintings in a row. Of these paintings, 2 are by Van Gogh, 5 are by Picasso, 3 by Monet (same as in Problem 2). This time you want to randomly select 2 paintings (without replacement) and study/research about them. What is the probability that you have at least one painting of Van Gogh?

Considering Van Gogh as "success" and the rest as "failure", let X be the random variable that represents number of successes when you draw 2 paintings.
$P(X \geq 1) = P(X = 1) + P(X = 2) = \frac{{}_2C_1 \times_8 C_1 +_2 C_2}{{}_{10}C2} = \frac{16+1}{45} \approx 0.3778$ **OR**

P(**at least one Gogh**) = P({**one Gogh and 1 Monet**}**or**{**one Gogh and 1 Picasso**} **or**{**two Gogh's paintings**}
$= \frac{{}_2C_1 \times_3 C_1 +_2 C_1 \times_5 C_1 +_2 C_2}{{}_{10}C_2} = \frac{6+10+1}{45} \approx 0.3778$

Problem IV. Be sure to show all work for full credit.

1. (6 points) Let X denote the diameter of an armored electric cable and Y denote the diameter of the ceramic mold that makes the cable. Both X and Y are scaled so that they range between 0 and 1. Suppose that X and Y have the joint density function

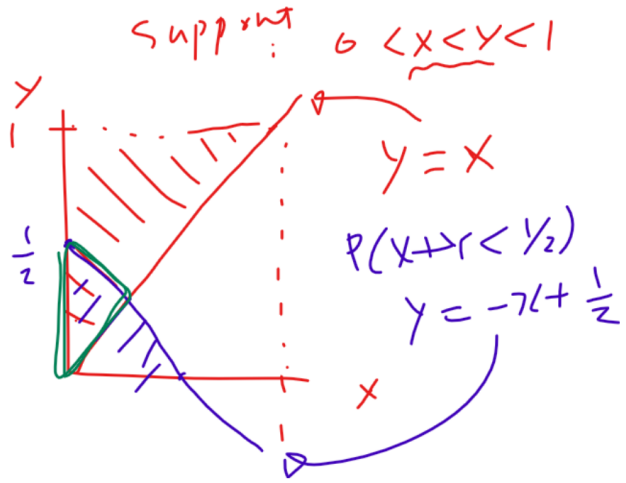
$$f(x,y) = \begin{cases} ky & \text{for } 0 < x < y < 1, \\ 0 & \text{elsewhere} \end{cases}$$

   (a) (3 points) Evaluate k.

$$\int_0^1 \int_0^y kydxdy = \int_0^1 ky(y-0)dy = \frac{k}{3}y^3\Big|_{y=0}^1 = \frac{k}{3}$$

must be 1. Hence $k = 3$.

   (b) (3 points) Assume that $k = 3$. Set up integrals to find $P(X + Y < 1/2)$ (You do not need to evaluate/perform the integration).



Integrate the density function over the green triangle.

$$\int_0^{1/4} \int_x^{-x+1/2} 3y dy dx$$

(optional) which results in $\frac{3}{64}$

It is also possible to have

$$\int_0^{1/4} \left[\int_0^{y} 3y dx \right] dy + \int_{1/4}^{1/2} \left[\int_0^{-y+1/2} 3y dx \right] dy$$

2. (6 points) Suppose a friend of yours has asked you to invest lots of money in a research venture examining a new cure for male pattern baldness. When you asked what the likelihood of success is, he says:

   - There is 30% chance that this research team will discover a cure that works. If the team does NOT find a cure, you will get $250.
   - Even if the searcher find a successful cure, there is only 60% chance the the U.S. Food and Drug Administration (FDA) will approve the new cure as safe for use on human. Once the cure is approved then it will become available in the market and the best estimate on your return is $25,000.
   - If the successful cure is not approved, you do not earn any money.

   What is expected value of your earning/return?

Let X be the random variable representing your earning/return.

$x = 250$ **if a new cure not discovered**
$x = 0$ **if a new cure discovered but not approved**
$x = 25000$ **if a new cure discovered and approved**
P(X=0) = P(discover and not approved) = (0.3)(0.4)=0.12
P(X=250)=P(not discover) = 0.7
P(X=25,000) = P(discover and approved) = P(discover)P(approved | discover)= (0.3)(0.6)=0.18
$E(X) = 0 \times 0.12 + 250 \times 0.7 + 25000 \times 0.18 = 4,675$

Problem V. Be sure to show all work for full credit.

1. (6 points) In a reliability context a randomly selected electronic component will undergo an accelerated failure time test. Suppose that
   - random variable X_1 takes the value 1 if the component lasts less than 50 hours and zero otherwise
   - random variable X_2 takes the value 1 if the component lasts between 50 to 90 hours and zero otherwise.
   - The probabilities that a randomly selected component will last less than 50 hours, between 50 and 90 hours and more than 90 hours are 0.2, 0.5, and 0.3, respectively.

   a) (3 points) Construct the joint probability distribution of X_1 and X_2.

$f(x,\ y)$		x_1		Row Total
		0	1	
x_2	0			
	1			
Column Total				

If a randomly selected component lasts less than 50 hours, then $x_1 = 1$ and $x_2 = 0$ with probability of 0.2
If a randomly selected component lasts between 50 and 90 hours, then $x_1 = 0$ and $x_2 = 1$ with probability of 0.5.
If a randomly selected component lasts more than 90 hours, then $x_1 = x_2 = 0$ with probability of 0.3.

f(x, y)		x_1		**Row Total**
		0	**1**	
x_2	**0**	**0.3**	**0.2**	**0.5**
	1	**0.5**	**0**	**0.5**
Column Total		**0.8**	**0.2**	**1**

   b) (3 points) Find $\rho(X_1, X_2)$, correlation between X_1 and X_2.

$Cov(X_1, X_2)$**.**
$Cov(X_1, X_2) = E(X_1X_2) - E(X_1)E(X_2)$
where $E(X_1X_2) = \sum (x_1x_2)P(X_1 = x_1, X_2 = x_2)$
$= (0\times0)(0.3)+(1\times0)(0.2)+(0\times1)(0.5)+(1\times1)(0) = 0$ **and** $E(X_1) = 0\times0.8+1\times0.2 = 0.2$
$E(X_2) = 0 \times 0.5 + 1 \times 0.5 = 0.5$
Hence
$Cov(X_1, X_2) = E(X_1X_2) - E(X_1)E(X_2) = 0 - (0.2)(0.5) = -0.1$