# Lab Handout Week 13

## Problem 1: What value of $z_{\alpha/2}$ or $t_{\alpha/2,df}$ is used to construct:

a. a 95% confidence interval to estimate $\mu_1 - \mu_2$, difference in two population means and two samples are independent. The sample size for sample 1 is 23 and for sample 2 is 41. (Assume random sample assumption and normal population distribution assumption are met and population variance unknown and unequal.)

```
> qt(1-0.05/2, df=23-1) = 2.073873
```

**we use conservative option for degrees of freedom (minimum of $n_1 - 1$ and $n_2 - 1$**

b. a 99% confidence interval to estimate $\mu_1 - \mu_2$, difference in two population means and two samples are independent. The sample size for sample 1 is 8 and for sample 2 is 10. (Assume random sample assumption and normal population distribution assumption are met and population variance unknown and equal.)

**Pool estimate for population variance// t-multiplier with degrees of freedom** $n_1 + n_2 - 2 = 8 + 10 - 2 = 16$

```
qt(1-0.01/2, df=16) = 2.92
```

c. a 93% confidence interval to estimate $\mu_D$ (mean of difference within pairs). There are 20 matched pairs.

```
qt(1-0.07/2, df=19) = 1.919992
```

d. a 98% confidence interval to estimate $\mu$ if the sample size is 1982. (Assume random sample assumption is met.)

```
qnorm(1-0.02/2) = 2.328232
```

**Note that for a large $n$, $t_{\alpha/2,df} \approx z_{\alpha/2}$**

## Problem 2. Two-sample Case Study

We consider a study involving a thorough graphical and formal analysis, and conclusion. In a data analysis study conducted by personnel at the Statistics Consulting Center at Virginia Tech, two different materials, alloy A and alloy B, were compared in terms of breaking strength. Alloy A is more expensive, but it should certainly be adopted if it can be shown to be stronger than alloy B. Random samples of beams made from each alloy were selected, and strength was measured in units of 0.001-inch deflection as a fixed force was applied at both ends of the beam. Twenty specimens were used for each of the two alloys. Use the following R commands to enter the data into R.

```
A<-c(88, 82, 87, 79, 85, 90, 84, 88, 83, 89, 80, 81, 81, 85, 83, 87, 82, 80, 79, 78)
B<-c(75, 81, 80, 77, 78, 81, 86, 78, 77,  84, 82, 78, 80, 80,  78, 76, 83, 85,  76, 79)
```

It is important that the engineer compare the two alloys. Of concern is average strength.

1. Construct graphical summaries (such as histogram, boxplot) to describe each distribution.

```
    par(mfrow=c(1, 2)) #one  by two frame
hist(A)
hist(B)
par(mfrow=c(1,1)) #back to one by one

#A : center around 82, 84, ranges fro 78 to 90, bimodal
#B : center around 80, ranges from 74 to 86, unimodal
# both no extreme skeweness



boxplot(A, B, names  =c("Alloy A", "Alloy B"), main="Strength of Alloy A and B")

#both distribution slightly skewed to the right,overall symmetric.
#A : middle 50\% of distribution is between 80 and 87
#B : middle 50\% of distribution between about 77 and 82
# range of two distribution seems not different =>use pooled estimate
```

2. Construct Q-Q normal plot to see if there is a severe violation of normality assumption.

```
par(mfrow=c(1, 2))
qqnorm(A)
qqline(A)
qqnorm(B)
qqline(B)
par(mfrow=c(1,1))
#both plots show tails don't follow
# straight line but no severe violation of normal assumption.
```

3. State the null and alternative hypothesis to test the claim that Alloy A is stronger than Alloy B.

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A > \mu_B$

**where $\mu_A$: mean strength of alloy A $\mu_B$: mean strength of alloy B.**

4. What type of test will you use?

**independent two sample t-test with unknown variance, equal variance (both distributions have similar range/variability based on boxplot, sample standard deviations)**

5. Use `t.test()` command to obtain test statistic and p-value.

```
> t.test(x=A, y=B, alternative="greater", var.equal=TRUE, conf.level=0.95)

Two Sample t-test

data:  A and B
t = 3.5895, df = 38, p-value =
0.0004671
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 2.041705      Inf
sample estimates:
mean of x mean of y
    83.55     79.70
```

**test statistic is $t^* = 3.5895$, if there was no difference in average strength between Alloy A and B, the observed difference in two samples $\bar{x}_A - \bar{x}_B = 83.55 - 79.70 = 3.85$ unit is 3.59 standard errors above the hypothesized difference (which is 0). This is unlikely. The data contradicts the null hypothesis.**

**$p$-value $= P(T_{38} > 3.5895) = 0.0004771$. If there was no difference in average strength between Alloy A and B, the probability of obtaining test statistic $t^* = 3.5895$ or more extreme is 0.0004671. This is extremely unlikely.**

6. Conclusion.

**We reject the null hypothesis at $\alpha = 0.05$. We have enough evidence to conclude that the average strength of Alloy A is statistically significantly greater than the average strength of Alloy B.**

7. Is the result practically significant?

**As the strength is measured in 0.0001-inch in deflection, the observed difference is 0.00385 inch ($\bar{x}_A - \bar{x}_B = 83.55 - 79.70 = 3.85$ in 0.0001 inch). As the magnitude of the difference is not big, hence the result is not practically significant. (the engineer must determine if this small difference is sufficient to justify the extra cost in the long run.).**

8. Construct a confidence interval to the difference between two population means and interpret.

```
    > t.test(x=A, y=B, alternative="two.sided", var.equal=TRUE, conf.level = .95)

Two Sample t-test

data:  A and B
t = 3.5895, df = 38, p-value =
0.0009342
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 1.678707 6.021293
sample estimates:
mean of x mean of y
    83.55     79.70
```

**95% confidence interval for $\mu_A - \mu_B$ is (1.6787, 6.0213).**

**We are 95% confident that the average strength for alloy A is between 1.67 and 6.02 units greater than the mean strength for alloy B.**