Lab handout

Stat 3021 Week 5 : Feb. 19, 2019

Chapter covered:

- Chapter 2.5 Addition rule
- Chapter 2.6 Conditional Probability, Independence, Multiplication rule
- Chapter 2.7 Law of Total Probability
- Chapter 3.1 Random variable
- Chapter 3.2 Discrete Probability Distributions
- chapter 3.3 Continuous Probability Distributions

1. A company has bid on two large construction projects. The company president believes that the probability of winning the first contract is 0.6, the probability of winning the second contract is 0.3, and the probability of winning neither is 0.2.

   (a) What is the probability that the company wins at least one contract?

   Let A: win the first, B: win the second. $P(A \cup B) = 1 - P(\text{winning neither}) = 0.8$

   (b) What is the probability that the company wins both contracts? $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.3 - 0.8 = 0.1$

   (c) what is the probability that the company wins the second contract given than they won the first one?
   $P(B|A) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.1}{0.6} = 1/6 \approx 0.1667$

2. A town has two fire engines operating independently. The probability that a specific engine is available when needed is 0.96.

   (a) What is the probability that neither is available when needed?
   $0.04^2 = 0.0016$

   (b) What is the probability that a fire engine is available when needed?
   $1 - 0.0016 = 0.9984$

3. If 7 cards are dealt from an ordinary deck of 52 playing cards, what is the probability that

   (a) exactly 2 of them will be face cards?

   $P(\text{2 face cards and 5 non-face cards}) = \dfrac{{}_{12}C_2 \times_{40} C_5}{{}_{52}C_7} = 0.3246$

   (b) at least 1 of them will be a queen? $P(\text{1, 2, 3, or 4 queens}) = 1 - P(\text{0 queen}) = 1 - \dfrac{{}_4C_0 \times_{48} C_7}{{}_{52}C_7} = 1 - 0.449644 = 0.5504$

4. Let X have the probability density function (pdf)

$$f(x) = \begin{cases} \frac{1}{2}x & 0 < x < k \\ 0 & elsewhere \end{cases}$$

   (a) Determine the value of k.

$$\int_0^k \frac{1}{2}x dx = \frac{1}{2}\left(\frac{x^2}{2}\right)\Big|_0^k = \frac{k^2}{4} = 1$$

   Hence k=2. (-2 doesn't work as it makes $f(x)$ negative.)

   (b) Find the cumulative distribution function $F(x)$.

$$\int_0^x \frac{1}{2}tdt = \frac{x^2}{4} \text{ for } 0 \leq x < 2$$

   Hence

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{x^2}{4} & 0 \leq x < 2 \\ 1 & \text{for } x \geq 2 \end{cases}$$

5. (Exercise 3.62 page 107) An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let X be the number of months between successive payments. The cumulative distribution function of X is

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.4 & \text{if } 1 \leq x < 3 \\ 0.6 & \text{if } 3 \leq x < 5 \\ 0.8 & \text{if } 5 \leq x < 7 \\ 1 & \text{if } x \geq 7 \end{cases}$$

(a) What is the probability mass function of X?
(Hint: $f(x) = F(x) - F(x-1)$ for discrete random variable.)

$f(x) = 0.4$ for $x = 1$, $f(x) = 0.2$ for $x = 3, 5, 7$, and $f(x) = 0$ elsewhere

(b) Find $P(4 < x \leq 7)$

$P(4 < x \leq 7) = F(7) - F(4) = 1 - 0.6 = 0.4$

6. (Exercise 3.39 modified, page 105) From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected.
(a) What is the total number of combinations?
${}_8C_4 = 70$

(b) Let X be the number of oranges and Y be the number of apples. What is $P(X = 1, Y = 1)$?

$\dfrac{{}_3C_1 \times {}_2C_1 \times {}_3C_2}{70} = 18/70 = 0.2571$

(c) Find $P(X + Y = 2)$
$P(X = 0, Y = 2) + P(X = 1, Y = 1) + P(X = 2, Y = 0) = 3/70 + 18/70 + 9/70 = 30/70 = 0.429$

7. Engie is an engineer for a popular social networking application. She builds a new emoji creation tool and wants to analyze user engagement. After analyzing the data collected on people using this tool, she discovers that the time in minutes until a person exits the emoji creator has a probability density function

$$f(x) = \begin{cases} \dfrac{1}{10} e^{-\frac{1}{10}x} & \text{for } x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Verify that $f(x)$ is a valid probability density function.

$f(x)$ is non-negative for all real number
Must show $\int_{-\infty}^{\infty} f(x)dx = 1$
$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} 0dx + \int_{0}^{\infty} \frac{1}{10} e^{-\frac{1}{10}x} dx = \int_{0}^{\infty} \frac{1}{10} e^{-\frac{1}{10}x} dx = -e^{-\frac{1}{10}x}\Big|_{x=0}^{\infty} = 1$

(b) Find the probability that a person uses the emoji creator for at least 2 minutes.

$P(X \geq 2) = -e^{-\frac{1}{10}x}\Big|_{x=2}^{\infty} = 0.8187$

(c) Find the probability that a person stops using the emoji creator within 10 minutes of use.

$P(0 \leq X \leq 10) = -e^{-\frac{1}{10}x}\Big|_{x=0}^{10} = 0.632$

(d) Find the probability that two randomly selected persons both stop using the emoji creator within 10 minutes of use.

For each user, the probability of stop using the emoji creator within 10 minutes is 0.632. Hence $0.632^2 = 0.399$