8.20

$X \sim f(x) = \begin{cases} \frac{1}{3} & x = 2, 4, 6 \\ 0 & \text{else} \end{cases}$ $\mu = \mathbb{E}X = \frac{1}{3}(2+4+6) = 4$

$\sigma^2 = \mathrm{Var}X = \frac{1}{3}(2^2+4^2+6^2) - (\mathbb{E}X)^2 = \frac{8}{3}$

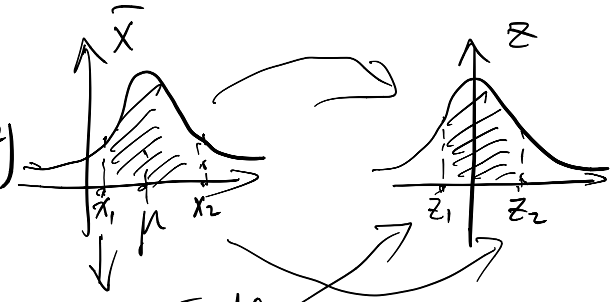
$X_1, \dots, X_n$, $n = 54$

Sample mean $\bar{X} = \frac{\sum_{i=1}^n X_i}{n} \underset{\text{CLT}}{\overset{\text{approx}}{\sim}} N(\mathbb{E}X, \frac{\mathrm{Var}X}{n}) = N(4, \frac{4}{81})$

(Remark. I use notation $N(\mu, \sigma^2)$, while your textbook use $N(\mu, \sigma)$.) So $\sigma_{\bar{x}} = \frac{2}{9}$.

$P(4.15 < \bar{X} < 4.35)$ $Z = \frac{\bar{X} - \mu}{\sigma} \sim N(0,1)$

$= P\left(\frac{0.15}{\sigma} < Z < \frac{0.35}{\sigma}\right)$



① $P(\bar{X} \le x)$ $\bar{X} \sim N(\mu, \sigma^2)$

$= \mathrm{pnorm}(x, \mu, \sigma)$

② $Z = \frac{\bar{X} - \mu}{\sigma} \sim N(0,1)$ $\frac{4.15 - 4}{2/9}$

$P(\bar{X} \le x) = P(Z \le z)$

$\underline{236} \in \mu_{\bar{x}} \pm 2\sigma_{\bar{x}}$

8.21

$X \sim P_X$ $n = 40$ $\mu_X = \mathbb{E}X = 240$, $\sigma_X = 15$

$\bar{X} \underset{\text{CLT}}{\sim} N(\mu_X, \frac{\sigma_X^2}{n})$ $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{15}{\sqrt{40}} \approx 2.37$

So $\bar{x} = 236$ is within $2\sigma_{\bar{x}}$ interval.

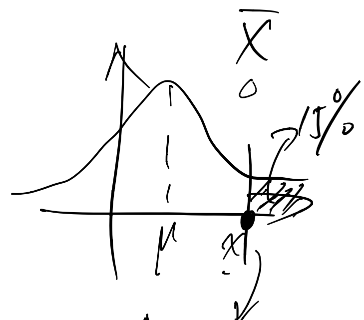
8.25 $X \sim N(7,1)$ $\mu_X = \mathbb{E}X = 7$, $\sigma_X^2 = \mathrm{Var}X = 1$

(a) $n=9$ $\bar{X} \overset{\text{exact}}{\sim} N(7, \frac{1}{9})$, $\sigma_{\bar{X}} = \frac{1}{3} = \frac{\sigma_X}{\sqrt{n}}$

(Remark: linear combination of normal distributions still follows normal. Don't rely on CLT.)

$P(6.8 < \bar{X} < 7.2) \Leftrightarrow P(\frac{-0.6}{\sigma} < Z < \frac{0.2}{\sigma})$

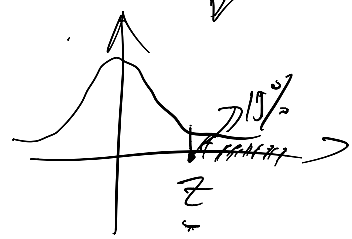
(b) $z = \text{qnorm}(0.85) \approx 1.04$



$z = \frac{x-\mu}{\sigma_{\bar{X}}} = \frac{x-7}{1/3}$

$x = \mu + z\sigma_{\bar{X}} \approx 7.35$

(Remark: $x = \text{qnorm}(0.85, \mu, \sigma_{\bar{X}})$)



8.29 (Compare of two semple means).

$X \sim (72, 10^2)$, $Y \sim (28, 5^2)$

$n_1 = 64$, $\bar{X} \sim N(72, \frac{10^2}{64})$ $n_2 = 100$, $\bar{Y} \sim N(28, \frac{5^2}{100})$

$P(\bar{X} - \bar{Y} < 44.2)$ $\quad U = \bar{X} - \bar{Y} \sim N(\mu_X - \mu_Y, \sigma_{\bar{X}}^2 + \sigma_{\bar{Y}}^2)$

$= N(44, \underbrace{\tfrac{29}{16}}_{\sigma^2})$

$= P(Z < \frac{0.2}{\sigma}) = P(U \leq 44.2)$