Outline:

1. Review of basic distributions
2. Review of expectation
3. Exercise from textbook.

1.

		Mean	Variance
Bernoulli	$Ber(p)$.	p	$p(1-p)$
Binomial	$Bin(n, p)$	np	$np(1-p)$
Poisson	$\mathcal{P}(\lambda)$	λ	λ
Uniform	$\mathcal{U}([a,b])$.	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential	$\mathcal{E}(\lambda)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal	$N(\mu, \sigma^2)$	μ	σ^2
Chi-square	χ^2_p	p	
(*) Gamma	$Gamma(\alpha, \beta)$		
student-t	t_p		
F	$F_{p,q}$		

$X_1, \cdots, X_n \overset{iid}{\sim} N(0,1)$

$Y_1, \cdots, Y_m \overset{iid}{\sim} N(0,1)$

$$X^2 = \sum_{i=1}^{n} X_i^2 \sim \chi^2_n$$

$$T = \frac{Y_1}{\sqrt{X^2/n}} \sim t_n.$$

$$Y^2 = \sum_{i=1}^{m} Y_i^2 \sim \chi^2_m$$

$$F = \frac{X^2/n}{Y^2/m} \sim F_{n,m}$$

2. Expectation of RV.

Assume X is continuous with pdf $f(x)$.

Def: $\mathbb{E}X = \int_{\mathbb{R}} x f(x)\,dx$, $\quad Var\,X = \mathbb{E}(X-\mathbb{E}X)^2 = \mathbb{E}X^2 - (\mathbb{E}X)^2$.

Prop: ① $\mathbb{E}(X+Y) = \mathbb{E}X + \mathbb{E}Y$

② $\mathbb{E}(aX) = a\mathbb{E}X$, $\forall a \in \mathbb{R}$.

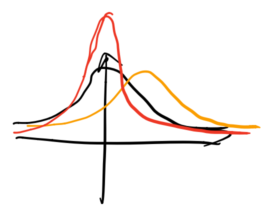
③ If $X \perp\!\!\!\perp Y$, $Var(X+Y) = Var\,X + Var\,Y$.

④ $Var(aX) = a^2 Var\,X$

e.g. $X_1, \cdots, X_n \overset{i.i.d}{\sim} N(\mu, \sigma^2)$, $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$,

then $\mathbb{E}\bar{X} = \mathbb{E}\frac{X_1+\cdots+X_n}{n} = \mu$.

$Var\,\bar{X} = Var\left(\frac{X_1+\cdots+X_n}{n}\right) = \frac{\sigma^2}{n}$.



A little bit more of Normal. Let $X \sim N(\mu, \sigma^2)$.

Then $\frac{X-\mu}{\sigma} \sim N(0,1)$.

Besides. Coro: $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$, $X \perp\!\!\!\perp Y$,

then $X+Y \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$.

3、 **7.15** Suppose that $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples, with the variables X_i normally distributed with mean μ_1 and variance σ_1^2 and the variables Y_i normally distributed with mean μ_2 and variance σ_2^2. The difference between the sample means, $\overline{X} - \overline{Y}$, is then a linear combination of $m + n$ normally distributed random variables and, by Theorem 6.3, is itself normally distributed.

a Find $E(\overline{X} - \overline{Y})$.

b Find $V(\overline{X} - \overline{Y})$.

c Suppose that $\sigma_1^2 = 2$, $\sigma_2^2 = 2.5$, and $m = n$. Find the sample sizes so that $(\overline{X} - \overline{Y})$ will be within 1 unit of $(\mu_1 - \mu_2)$ with probability .95.

$$\bar{X} \sim N(\mu_1, \frac{\sigma_1^2}{m}) \quad \bar{Y} \sim N(\mu_2, \frac{\sigma_2^2}{n})$$

So $$\bar{X} - \bar{Y} \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}) \quad \left(-\bar{Y} \sim N(-\mu_2, \frac{\sigma_2^2}{n})\right)$$

$$P\left(\left| (\bar{X} - \bar{Y}) - (\mu_1 - \mu_2) \right| < 1 \right) = 0.95.$$

$$\Leftrightarrow P\left(\frac{|(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)|}{\sqrt{\sigma_1^2/n + \sigma_2^2/n}} < \frac{1}{\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}} \right) = 0.95$$

$$Z \overset{def}{=} \frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n + \sigma_2^2/n}} \sim N(0, 1)$$

$$\therefore \frac{1}{\sqrt{\frac{\sigma_1^2 + \sigma_2^2}{n}}} = Z_{0.025} = 1.96$$