Outline:

① Review of last lab

② Exercise

7.21 Refer to Exercise 7.13. Suppose that $n = 20$ observations are to be taken on ln(LC50) measurements and that $\sigma^2 = 1.4$. Let S^2 denote the sample variance of the 20 measurements.

- **a** Find a number b such that $P(S^2 \le b) = .975$.
- **b** Find a number a such that $P(a \le S^2) = .975$.
- **c** If a and b are as in parts (a) and (b), what is $P(a \le S^2 \le b)$?

$X \sim N(\mu, \sigma^2)$, $\sigma^2 = 1.4$ $\quad X_i, i = 1, \cdots, n \overset{i.i.d}{\sim} X$.

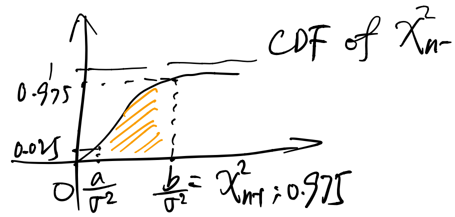
$$S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1} \sim \sigma^2 \chi^2_{n-1}, \quad \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}.$$

(a) $P(\sigma^2 \chi^2_{n-1} \le b) = 0.975 \Leftrightarrow P(\chi^2_{n-1} \le \frac{b}{\sigma^2}) = 0.975$

$\frac{b}{\sigma^2} = \chi^2_{n-1;\,0.975}$ Notation X, α $\quad X_\alpha \equiv \alpha$-quantile of X.

(b) $\frac{a}{\sigma^2} = \chi^2_{n-1;\,0.025}$

(c) $P(a \le S^2 \le b) = 0.95$



***7.30** Suppose that Z has a standard normal distribution and that Y is an independent χ^2-distributed random variable with ν df. Then, according to Definition 7.2,

$$T = \frac{Z}{\sqrt{Y/\nu}}$$

has a t distribution with ν df.[1]

b According to the result derived in Exercise 4.112(a), if Y has a χ^2 distribution with ν df, then

$$E(Y^a) = \frac{\Gamma([\nu/2] + a)}{\Gamma(\nu/2)} 2^a, \qquad \text{if } \nu > -2a.$$

Use this result, the result from part (a), and the structure of T to show the following. [*Hint:* Recall the independence of Z and Y.]

- **i** $E(T) = 0$, if $\nu > 1$.
- **ii** $V(T) = \nu/(\nu - 2)$, if $\nu > 2$.

(i) $\mathbb{E}T = \mathbb{E}\frac{Z}{\sqrt{Y/\nu}} = \mathbb{E}Z \cdot \mathbb{E}\frac{1}{\sqrt{Y/\nu}}$ $(\mathbb{E}Z=0)$

$$= 0$$

(ii) $Var\,T = \mathbb{E}T^2 - (\mathbb{E}T)^2 = \mathbb{E}T^2$

$$= \mathbb{E}\frac{Z^2}{Y/\nu} = \nu\,\mathbb{E}\frac{Z^2}{Y} = \nu\,\mathbb{E}Z^2\,\mathbb{E}Y^{-1}$$

$\mathbb{E}Z^2 = 1$, $\mathbb{E}Y^{-1} = \frac{\Gamma(\frac{\nu}{2}-1)}{\Gamma(\frac{\nu}{2})}2^{-1} = \frac{1}{\frac{\nu}{2}-1}\cdot\frac{1}{2} = \frac{1}{\nu-2}$.

$(\Gamma(\alpha+1) = \Gamma(\alpha)\cdot\alpha)$

7.43 An anthropologist wishes to estimate the average height of men for a certain race of people. If the population standard deviation is assumed to be 2.5 inches and if she randomly samples 100 men, find the probability that the difference between the sample mean and the true population mean will not exceed .5 inch.

7.44 Suppose that the anthropologist of Exercise 7.43 wants the difference between the sample mean and the population mean to be less than .4 inch, with probability .95. How many men should she sample to achieve this objective?

X, $\mathbb{E}X = \mu$, $\sigma^2 = Var\,X = 2.5^2$ $n=100$

$P(|\bar{X}-\mu| < 0.5)$

By CLT, $\frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}} \sim N(0,1)$ $Z \sim N(0,1)$

$P(|\bar{X}-\mu| < 0.4) = 0.95$

$\Leftrightarrow P\left(Z < \frac{0.4}{\sqrt{\sigma^2/n}}\right) = 0.95 \Rightarrow \frac{0.4}{\sqrt{\sigma^2/n}} = Z_{0.95}$

7.73 An airline finds that 5% of the persons who make reservations on a certain flight do not show up for the flight. If the airline sells 160 tickets for a flight with only 155 seats, what is the probability that a seat will be available for every person holding a reservation and planning to fly?

$X = \begin{cases} 1 & \text{show up} \\ 0 & \end{cases}$ $P(X=1) = 0.95$

$X_1, \dots, X_{160} \overset{i.i.d.}{\sim} X$, $n=160$

$P(\sum_{i=1}^{n} X_i \le 155)$ $\quad$ $X \sim Ber(p=0.95)$

$\mu = \mathbb{E}X = p$, $\sigma^2 = Var X = p(1-p)$.

$\frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}} \sim N(0,1)$

$P(\sum_{i=1}^{n} X_i \le 155) = P\left(\frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}} \le \frac{\frac{155}{n}-\mu}{\sqrt{\sigma^2/n}} \right)$

$= P(Z \le C)$.

Example of CLT.

Alice. $X_0 = a$, $a>0$.

$X_i \overset{iid}{\sim} X$ $\quad$ $P(X=1) = p$, $P(X=-1) = 1-p$

$\mathbb{E}X = 2p-1$, $Var X = 4p(1-p) = \sigma^2$

$S_n = a + \sum_{i=1}^{n} X_i$

$P(S_n \le 0) = P(\sum_{i=1}^{n} X_i \le -a)$

$= P\left(\frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}} \le \frac{-\frac{a}{n}-\mu}{\sqrt{\sigma^2/n}} \right)$.

If $p = 0.49$, $\frac{\frac{a}{n}-\mu}{\sqrt{\sigma^2/n}} = \frac{-\frac{a}{n}+0.02}{\sqrt{1/n}} = C_n$

when n is large, $C_n \approx 0.02\sqrt{n}$

$P(S_n \le 0) \approx P(Z \le 0.02\sqrt{n})$

Alice will stop if $\quad S_n = 0$ or $b > a$

P(Alice bankrupt)?